Half life

Half life is a type of exponential decay. Specifically, half life is the amount of time required for exactly half of our original substance to decay, leaving exactly half of what we started with. The decay occurs at a constant exponential rate, but the rate is different for every substance.

Scientists use half life to calculate how long it will take for specific substances to partially or completely break down. As an example, since we know that radioactive elements decay at an exponential rate, we can use half life to calculate their decay and see how long it'll take for the element to reach safe levels. Half life is also used to calculate the age of extremely old objects.

The basic equation for exponential decay is

$$y = Ce^{kt}$$

where C is the amount we started with, k is the decay constant, and y is the amount we have left over after time t. Since substances decay at different rates, k will be different depending on the substance.

When we're talking about half life, we know that

$$y = \frac{C}{2}$$

because we're always interested in how long it takes until we have exactly half of the amount of substance we started with originally. When we know C and t, we can substitute into the basic decay equation and get

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$$\frac{C}{2} = Ce^{kt}$$

Example

Fermium-253 has a half life of 3 days. If we start with 1,200 mg, how much Fermium-253 will be left after 10 days?

First we need to find k. Since we know that $C = 1{,}200$ and that $t = 3$, we can use

$$\frac{C}{2} = Ce^{kt}$$

$$\frac{1{,}200}{2} = 1{,}200e^{k(3)}$$

Solving the equation for k, we get

$$\frac{1}{2} = e^{3k}$$

$$\ln\frac{1}{2} = \ln e^{3k}$$

$$\ln 0.5 = 3k$$

$$k = \frac{\ln 0.5}{3}$$

Now that we have a value for k, we can solve for y using $y = Ce^{kt}$, where $C = 1{,}200$, $t = 10$ and $k = \frac{\ln 0.5}{3}$.

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$$y = 1{,}200e^{\frac{\ln 0.5}{3}(10)}$$

$$y = 1{,}200e^{\frac{10}{3}\ln 0.5}$$

$$y = 119.06 \text{ mg}$$

There would be 119.06 mg of Fermium-253 left after 10 days.

Let's try a more complex example.

Example

The half life of Americium-243 is $7{,}370$ years. How long will it take a mass of Americium-243 to decay to $73\,\%$ of its original size?

First we need to find k. We can assume that the original mass is $100\,\%$ of its original size and say that $C = 1$, and we already know that $t = 7{,}370$, so we can use

$$\frac{C}{2} = Ce^{kt}$$

to find k.

$$\frac{1}{2} = 1e^{k(7{,}370)}$$

$$\frac{1}{2} = e^{7{,}370k}$$

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$$\ln \frac{1}{2} = \ln e^{7{,}370k}$$

$$\ln \frac{1}{2} = 7{,}370k$$

$$k = \frac{\ln 0.5}{7{,}370}$$

Now that we have k, we can find t using $y = Ce^{kt}$, where $C = 1$ and $y = 0.73$.

$$0.73 = 1e^{\frac{\ln 0.5}{7{,}370}t}$$

$$\ln 0.73 = \ln e^{\frac{\ln 0.5}{7{,}370}t}$$

$$\ln 0.73 = \frac{\ln 0.5}{7{,}370}t$$

$$t = \frac{7{,}370 \ln 0.73}{\ln 0.5}$$

$$t = 3{,}346.21 \text{ years}$$

It would take $3{,}346.21$ years for our sample to decay to $73\,\%$ of its original size.

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